David versus Goliath:Small Cells versus Massive MIMO

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Supélec

Alcatel·Lucent

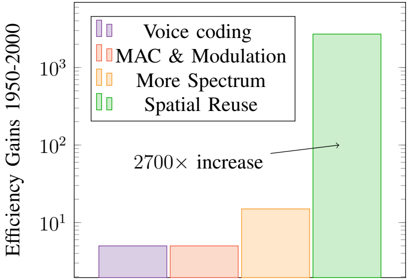
"The total worldwide mobile traffic is expected to increase 33× from 2010–2020.[1]"

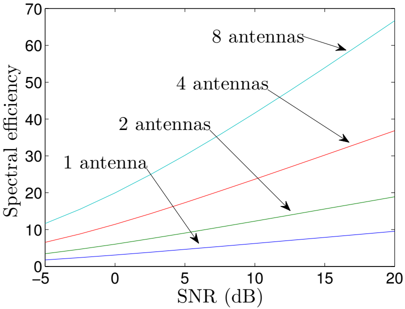
"The average 3G smart phone user consumed 375 MB/month. The average 3G broadband (HSPA/+) user consumed 5 GB/month. The average LTE consumer used 14–15 GB/month of data.[2]"

[1] Source: IDATE for UMTS Forum

[2] Press release of a Scandinavian operator (Nov. 2010)

Network densification

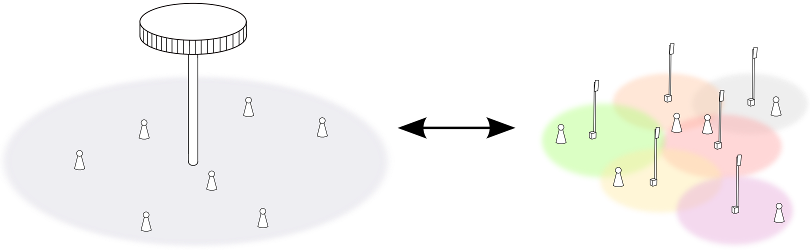




The exploding demand for wireless data traffic requires a massive network densification:

> Densification: *"Increasing the number of antennas per unit area"*

“David vs Goliath“ or ”Small Cells vs Massive MIMO“

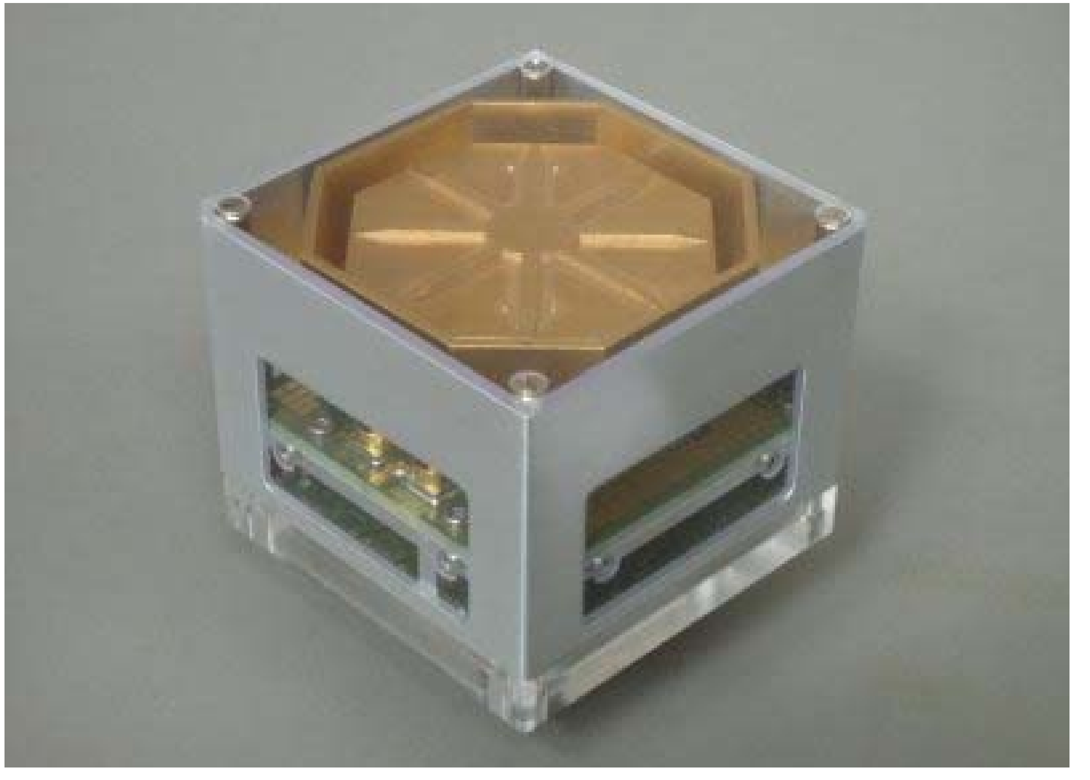


How to densify: *“More antennas or more BSs?”*

Questions:

- Should we install more base stations or simply more antennas per base?
- How can massively many antennas be efficiently used?
- Can massive MIMO simplify the signal processing?

Vision



Bell Labs lightradio antenna module – the next generation small cell
(picture from www.washingtonpost.com)

A thought experiment

Consider an infinite large network of randomly uniformly distributed base stations and user terminals.

What would be better?

A $2 \times$ more base stations

B $2 \times$ more antennas per base station

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Stochastic geometry can provide an answer.

System model: Downlink

Received signal at a tagged UT at the origin:

$$y = \underbrace{\frac{1}{r_0^{\alpha/2}} \mathbf{h}_0^{\mathsf{H}} \mathbf{x}_0}_{\text{desired signal}} + \underbrace{\sum_{i=1}^{\infty} \frac{1}{r_i^{\alpha/2}} \mathbf{h}_i^{\mathsf{H}} \mathbf{x}_i}_{\text{interference}} + n$$

- $\mathbf{h}_i \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$: fast fading channel vectors
- r_i: distance to ith closest BS
- $P = \mathbb{E}\left[\mathbf{x}_i^{\mathsf{H}} \mathbf{x}_i\right]$: average transmit power constraint per BS

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Assumptions:

- infinitely large network of uniformly randomly distributed BSs and UTs with densities λ_{BS} and λ_{UT}, respectively
- single-antenna UTs, N antennas per BS
- each UT is served by its *closest* BS
- distance-based path loss model with path loss exponent $\alpha > 2$
- total bandwidth W, re-used in each cell

Transmission strategy: Zero-forcing

Assumptions:

- $\mathcal{K} = \frac{\lambda_{\text{UT}}}{\lambda_{\text{BS}}}$ UTs need to be served by each BS on average
- total bandwidth W divided into $L \geq 1$ sub-bands
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Transmit vector of BS i:

$$\mathbf{x}_i = \sqrt{\frac{P}{K}} \sum_{k=1}^{K} \mathbf{w}_{i,k} s_{i,k}$$

- $s_{i,k} \sim \mathcal{CN}(0,1)$: message determined for UT k from BS i
- $\mathbf{w}_{i,k} \in \mathbb{C}^{N \times 1}$: ZF-beamforming vectors

Performance metric: Average throughput

Received SINR at tagged UT:

$$\gamma \;=\; \frac{r_0^{-\alpha}\left|\mathbf{h}_0^{\mathrm{H}}\mathbf{w}_{0,1}\right|^2}{\sum_{i=1}^{\infty} r_i^{-\alpha}\sum_{k=1}^{K}\left|\mathbf{h}_i^{\mathrm{H}}\mathbf{w}_{i,k}\right|^2+\frac{K}{P}} \;=\; \frac{r_0^{-\alpha}S}{\sum_{i=1}^{\infty} r_i^{-\alpha}g_i+\frac{K}{P}}$$

Coverage probability:

$$P_{\text{cov}}(T)=\mathbb{P}\left(\gamma \geq T\right)$$

Average throughput per UT:

$$\boxed{C \;=\; \frac{W}{L}\times\mathbb{E}\left[\log(1+\gamma)\right]=\frac{W}{L}\times\int_0^{\infty}P_{\text{cov}}\left(e^{z}-1\right)dz}$$

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Remarks:

- expectation with respect to fading *and* BSs locations
- $S=\left|\mathbf{h}_0^{\mathsf{H}}\mathbf{w}_{0,1}\right|^2\sim\Gamma(N-K+1,1)$, $\qquad g_i=\sum_{k=1}^{K}\left|\mathbf{h}_i^{\mathsf{H}}\mathbf{w}_{i,k}\right|^2\sim\Gamma(K,1)$
- K impacts the interference distribution, N impacts the desired signal
- for $P\to\infty$, the SINR becomes independent of λ_{BS}

A closed-form result

Theorem (Combination of Baccelli'09, Andrews'10)

$$P_{cov}(T) = \int_{r_0>0} \int_{-\infty}^{\infty} \mathcal{L}_{I_{r_0}}\left(i2\pi r_0^{\alpha} T s\right) \exp\left(-\frac{i2\pi r_0^{\alpha} TK}{P} s\right) \frac{\mathcal{L}_S\left(-i2\pi s\right) - 1}{i2\pi s} f_{r_0}(r_0) ds dr_0$$

where

$$\mathcal{L}_{I_{r_0}}(s) = \exp\left(-2\pi\lambda_{BS} \int_{r_0}^{\infty} \left(1 - \frac{1}{\left(1 + sv^{-\alpha}\right)^K}\right) vdv\right)$$

$$\mathcal{L}_S(s) = \left(\frac{1}{1+s}\right)^{N-K+1}$$

$$f_{r_0}(r_0) = 2\pi\lambda_{BS} r_0 e^{-\lambda_{BS}\pi r_0^2}$$

The computation of $P_{cov}(T)$ requires in general three numerical integrals.

J. G. Andrews, F. Baccelli, R. K. Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks" IEEE Trans. Wireless Commun., submitted 2010.

F. Baccelli, B. Błaszczyszyn, P. Mühlethaler, "Stochastic Analysis of Spatial and Opportunistic Aloha" Journal on Selected Areas in Communications, 2009

Example

- Density of UTs: $\lambda_{\mathrm{UT}} = 16$
- Constant transmit power density: $P \times \lambda_{\mathrm{BS}} = 10$
- Number of BS-antennas: $N = \lambda_{\mathrm{UT}} / \lambda_{\mathrm{BS}}$
- Path loss exponent: $\alpha = 4$
- UT simultaneously served on each band: $K = \lambda_{UT} / (\lambda_{\mathrm{BS}} \times L)$

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Table: Average spectral efficiency C/W in (bits/s/Hz)

sub-bands L	$\lambda_{\text{BS}} = 1$	$\lambda_{\text{BS}} = 2$	$\lambda_{\text{BS}} = 4$	$\lambda_{\text{BS}} = 8$	$\lambda_{\text{BS}} = 16$
1	0.6209	0.8188	1.1964	1.5215	2.1456
2	1.1723	1.2414	1.3404	1.5068	x
4	0.8882	0.8973	1.1964	x	x
8	0.5689	0.5952		x	x
16	0.3532	x	x	x	x

Fully distributing the antennas gives highest throughput gains!

First conclusions

- Distributed network densification is preferable over massive MIMO if the average throughput per UT should be increased.
- More antennas increase the coverage probability, but more BSs lead to a linear increase in area spectral efficiency (with constant total transmit power).
- If we use other metrics such as coverage probability or goodput, the picture might change.

Cellular Dreams and Cordless Nightmares

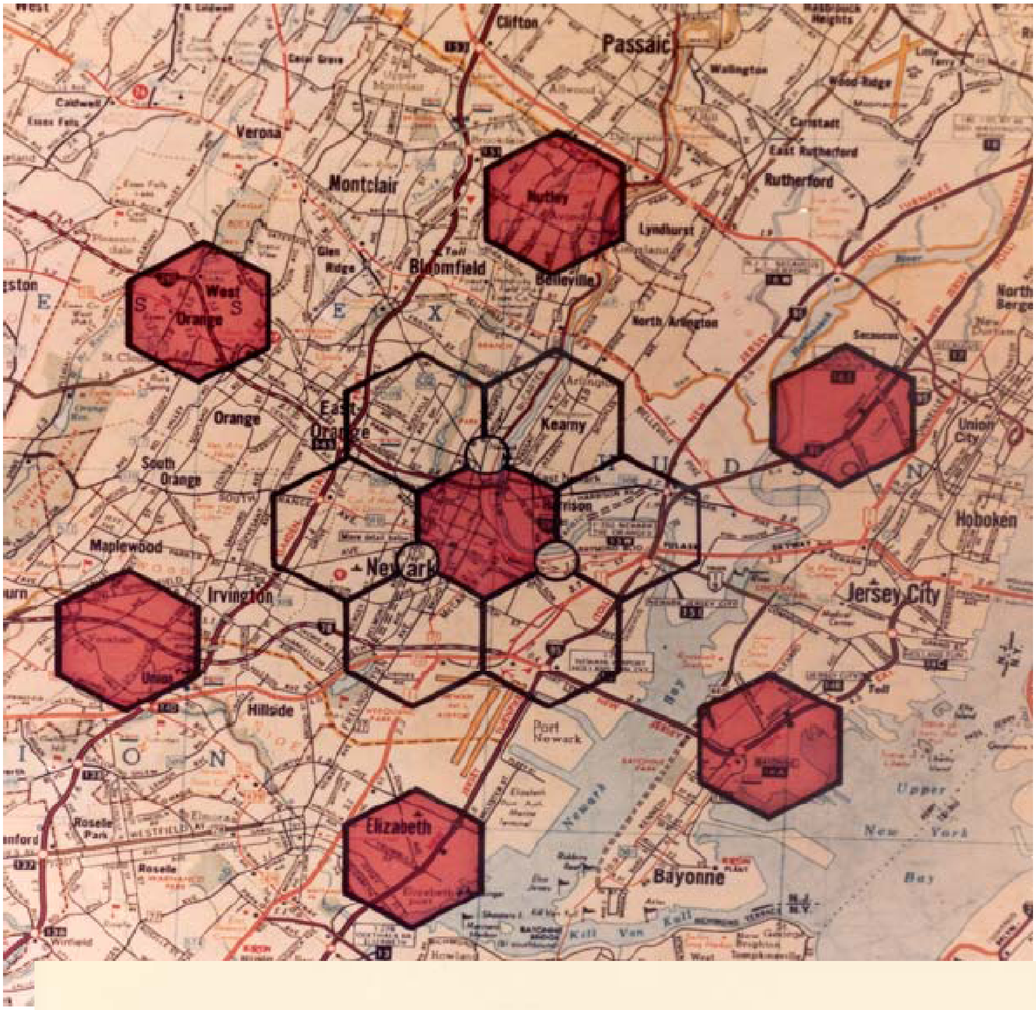
Life at Bell Laboratories in Interesting Times

Richard H. Frenkiel

Trials and Tribulations

By 1976, the time had come to prove that our many claims could be turned into a practical system. Small cell coverage over a large service area would require hundreds of cells and cost hundreds of millions of dollars, so we applied for permission to conduct two separate trials. A large-cell Market Trial in Chicago would provide realistic service to more than 2000 customers, while a small-cell "Test Bed" in Newark, New Jersey, would demonstrate that the smallest cells could provide good service in the presence of nearby interference. In combination, these trials would provide a complete demonstration of our system.

Motorola objected to our proposal as inadequate, since neither the trial in Chicago nor the Test Bed in Newark demonstrated a fully developed small-cell system. Chicago, they argued, used very large cells, while Newark was only a partial grid of small cells. Since a demonstration of small cells over a large area was clearly impractical, we were confident that the FCC would see Motorola's objections for what they were—another smoke screen intended to delay progress. As it turned out, our faith was misplaced. The FCC ruled that our proposed trials were inadequate, using virtually the same arguments that Motorola had presented, and summarily denied our application.



The partial small-cell grid in Newark and the Test Van

Infinitely Many Antennas: Forward-Link Capacity For 20 MHz Bandwidth, 42 Terminals per Cell, 500 μsec Slot

Interference-limited: energy-per-bit can be made arbitrarily small!

Frequency Reuse	.95-Likely SIR (dB)	.95-Likely Capacity per Terminal (Mbits/s)	Mean Capacity per Terminal (Mbits/s)	Mean Capacity per Cell (Mbits/s)
1	-29	.016	44	1800
3	-5.8	.89	28	1200
7	8.9	3.6	17	730

				Mean Capacity per Cell (Mbits/s)
LTE Advanced (>= Release 10)				74

Motivation of massive MIMO

Consider a $N \times K$ MIMO MAC:

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k x_k + \mathbf{n}$$

where $\mathbf{h}_k, \mathbf{n}$ are i.i.d. with zero mean and unit variance.

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By the strong law of large numbers:

$$\frac{1}{N} \mathbf{h}_m{}^{\mathsf{H}} \mathbf{y} \xrightarrow[N\to\infty,\ K=\text{const.}]{\text{a.s.}} x_m$$

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With an unlimited number of antennas,

- uncorrelated interference and noise vanish,
- the matched filter is optimal,
- the transmit power can be made arbitrarily small.

T. Marzetta, "Noncooperative cellular wireless with unlimited numbers of base station antennas" IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 35903600, Nov. 2010.

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  What does *small* mean?
- The channel provides infinite diversity, i.e., each antenna gives an independent look on the transmitted signal.
  What if the degrees of freedom are limited?
- The received energy grows without bounds as $N \to \infty$.
  Clearly wrong, but might hold up to very large antenna arrays if the aperture scales with N.

On channel estimation and pilot contamination

1. The receiver estimates the channels based on pilot sequences.
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With an unlimited number of antennas,

- uncorrelated interference, noise and estimation errors vanish,
- the matched filter is optimal,
- the transmit power can be made arbitrarily small ($\sim 1/\sqrt{N}$ [Ngo'11]),
- but the performance is limited by pilot contamination.

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Uplink

System model and channel estimation

Uplink: L BSs with N antennas, K UTs per cell. Received signal at BS j:

$$\mathbf{y}_j = \sqrt{\rho} \sum_{l=1}^{L} \mathbf{H}_{jl} \mathbf{x}_l + \mathbf{n}_j$$

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The columns of $\mathbf{H}_{jl}$ ($N \times K$) are modeled as

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$$\hat{\mathbf{h}}_{jjk} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Phi}_{jjk}\right), \qquad \tilde{\mathbf{h}}_{jjk} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{jjk} - \mathbf{\Phi}_{jjk}\right)$$

$$\mathbf{\Phi}_{jlk} = \mathbf{R}_{jjk} \mathbf{Q}_{jk} \mathbf{R}_{jlk}, \quad \mathbf{Q}_{jk} = \left(\frac{1}{\rho_\tau} \mathbf{I}_N + \sum_{l} \mathbf{R}_{jlk} \right)^{-1}$$

Achievable rates with linear detectors

Ergodic achievable rate of UT m in cell j:

$$R_{jm} = \mathbb{E}_{\hat{\mathbf{H}}_{jj}}\left[\log_2\left(1+\gamma_{jm}\right)\right]$$

$$\gamma_{jm} = \frac{\left|\mathbf{r}_{jm}^{\mathsf{H}}\hat{\mathbf{h}}_{jjm}\right|^2}{\mathbb{E}\left[\mathbf{r}_{jm}^{\mathsf{H}}\left(\frac{1}{\rho}\mathbf{I}_N + \tilde{\mathbf{h}}_{jjm}\tilde{\mathbf{h}}_{jjm}^{\mathsf{H}} - \mathbf{h}_{jjm}\mathbf{h}_{jjm}^{\mathsf{H}} + \sum_l \mathbf{H}_{jl}\mathbf{H}_{jl}^{\mathsf{H}}\right)\mathbf{r}_{jm}\ \middle|\hat{\mathbf{H}}_{jj}\right]}$$

with an arbitrary receive filter $\mathbf{r}_{jm}$.

B. Hassibi and B. M. Hochwald, "How much training is needed in multiple-antenna wireless links?" IEEE Trans. Inf. Theory., vol. 49, no. 4, pp. 951–963, Nov. 2003.

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Two specific linear detectors $\mathbf{r}_{jm}$:

$$\mathbf{r}_{jm}^{\mathsf{MF}} = \hat{\mathbf{h}}_{jjm}$$

$$\mathbf{r}_{jm}^{\mathsf{MMSE}} = \left(\hat{\mathbf{H}}_{jj}\hat{\mathbf{H}}_{jj}^{\mathsf{H}} + \mathbf{Z}_j + N\lambda\mathbf{I}_N\right)^{-1}\hat{\mathbf{h}}_{jjm}$$

where $\lambda > 0$ is a design parameter and

$$\mathbf{Z}_j = \mathbb{E}\left[\tilde{\mathbf{H}}_{jj}\tilde{\mathbf{H}}_{jj}^{\mathsf{H}} + \sum_{l\neq j}\mathbf{H}_{jl}\mathbf{H}_{jl}\right] = \sum_k\left(\mathbf{R}_{jjk} - \mathbf{\Phi}_{jjk}\right) + \sum_{l\neq j}\sum_k \mathbf{R}_{jlk}.$$

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Large system analysis based on random matrix theory

Assume $N, K \to \infty$ at the same speed. Then,

$$\gamma_{jm} - \bar{\gamma}_{jm} \xrightarrow{\text{a.s.}} 0$$

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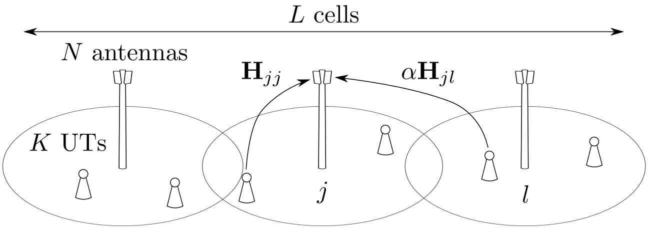
where

$$\bar{\gamma}_{jm}^{\text{MF}} = \frac{\left(\frac{1}{N}\operatorname{tr}\boldsymbol{\Phi}_{jjm}\right)^2}{\frac{1}{\rho N^2}\operatorname{tr}\boldsymbol{\Phi}_{jjm} + \frac{1}{N}\sum_{l,k}\frac{1}{N}\operatorname{tr}\mathbf{R}_{jlk}\boldsymbol{\Phi}_{jjm} + \sum_{l\neq j}\left|\frac{1}{N}\operatorname{tr}\boldsymbol{\Phi}_{jlm}\right|^2}$$

$$\bar{\gamma}_{jm}^{\text{MMSE}} = \frac{\delta_{jm}^2}{\frac{1}{\rho N^2}\operatorname{tr}\boldsymbol{\Phi}_{jjm}\bar{\mathbf{T}}_j' + \frac{1}{N}\sum_{l,k}\mu_{jlkm} + \sum_{l\neq j}|\vartheta_{jlm}|^2}$$

and δ_{jm}, μ_{jlkm}, θ_{jlm}, $\bar{\mathbf{T}}_j'$ can be calculated numerically.

A simple multi-cell scenario



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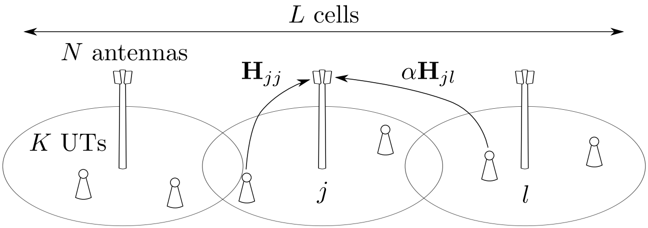

- intercell interference factor $\alpha \in [0, 1]$
- transmit power per UT: ρ
- $\mathbf{H}_{jl} = [\mathbf{h}_{jl1} \cdots \mathbf{h}_{jlK}] = \sqrt{N/P}\mathbf{A}\mathbf{W}_{jl}$
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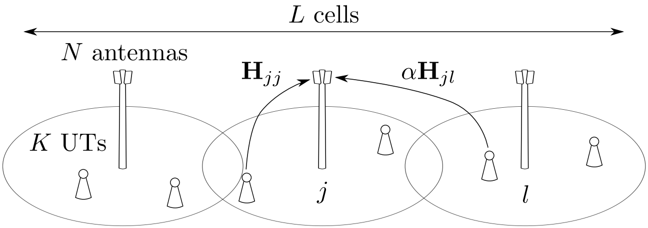

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Assumptions:

- P channel degrees of freedom, i.e., $\text{rank}\left(\mathbf{H}_{jl}\right) = \min(P, K)$ [Ngo'11]
- energy scales linearly with N, i.e., $\mathbb{E}\left[\text{tr}\, \mathbf{H}_{jl}\mathbf{H}_{jl}^{\text{H}}\right] = KN$
- only pilot contamination, i.e., no estimation noise:

$$\hat{\mathbf{h}}_{jjk} = \mathbf{h}_{jjk} + \sqrt{\alpha} \sum_{l \neq j} \mathbf{h}_{jlk}$$

Asymptotic performance of the matched filter

Assume that N, K and P grow infinitely large at the same speed:

$$\mathrm{SINR}^{\mathrm{MF}} \approx \frac{1}{\underbrace{\frac{\bar{L}}{\rho N}}_{\text{noise}} + \underbrace{\frac{K}{P}\bar{L}^2}_{\text{multi-user interference}} + \underbrace{\alpha(\bar{L}-1)}_{\text{pilot contamination}}}$$

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Observations:

- The effective SNR ρN increases linearly with N.
- The multiuser interference depends on P/K and not on N.
- Ultimate performance limit:

$$\mathsf{SINR}^{\mathsf{MF}} \xrightarrow[N,P\to\infty,\ K=\text{const.}]{\text{a.s}} \mathsf{SINR}^{\infty} = \frac{1}{\alpha(\bar{L}-1)}$$

J. Hoydis, S. ten Brink, and M. Debbah, "Massive MIMO: How many antennas do we need", Allerton Conference, Urbana-Champaing, Illinois, US, Sep. 2011. [Online] http://arxiv.org/abs/1107.1709

Asymptotic performance of the MMSE detector

Assume that N, K and P grow infinitely large at the same speed:

$$\mathsf{SINR}^{\mathsf{MMSE}} \approx \frac{1}{\underbrace{\frac{\bar{L}}{\rho N} X}_{\text{noise}} + \underbrace{\frac{K}{P} \bar{L}^2 Y}_{\text{multi-user interference}} + \underbrace{\alpha(\bar{L} - 1)}_{\text{pilot contamination}}}$$

where $\bar{L} = 1 + \alpha(L - 1)$ and X, Y are given in closed-form.

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Observations:

- As for the MF, the performance depends only on ρN and P/K.
- The ultimate performance of MMSE and MF coincide:

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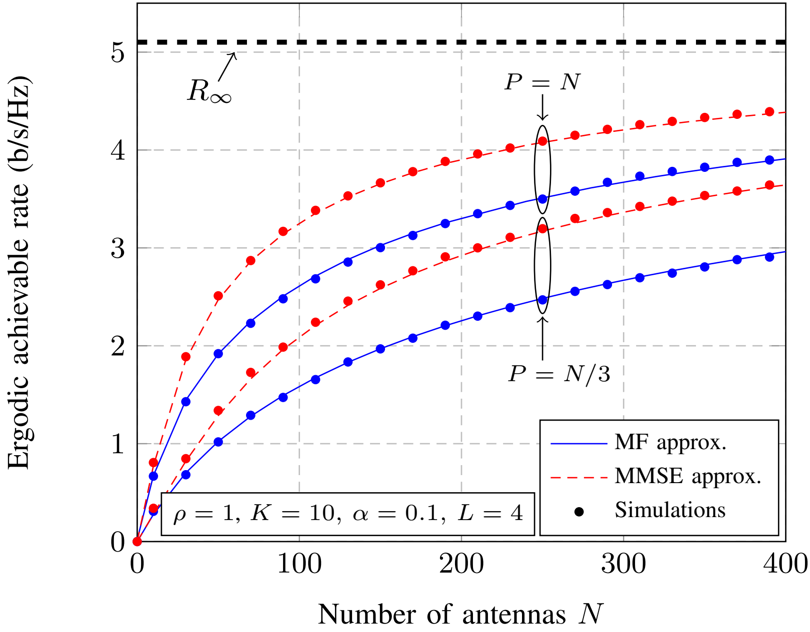

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- The number of antennas needed for massive MIMO depends on *all* these parameters!

Downlink

System model: Downlink

L BSs with N antennas, K UTs per cell. Received signal at mth UT in cell j:

$$y_{jm} = \sqrt{\rho} \sum_{l=1}^{L} \mathbf{h}_{ljm}^{\mathsf{H}} \mathbf{s}_l + q_{jm}$$

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Channel estimation through uplink pilots (as before):

$$\mathbf{h}_{jjk} = \hat{\mathbf{h}}_{jjk} + \tilde{\mathbf{h}}_{jjk}$$

$$\hat{\mathbf{h}}_{jjk} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{\Phi}_{jjk}\right), \qquad \tilde{\mathbf{h}}_{jjk} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_{jjk} - \mathbf{\Phi}_{jjk}\right)$$

$$\mathbf{\Phi}_{jlk} = \mathbf{R}_{jjk} \mathbf{Q}_{jk} \mathbf{R}_{jlk}, \quad \mathbf{Q}_{jk} = \left(\frac{1}{\rho_\tau} \mathbf{I}_N + \sum_l \mathbf{R}_{jlk}\right)^{-1}$$

Achievable rates with linear precoders

Ergodic achievable rate of UT m in cell j:

$$R_{jm} = \log_2\left(1 + \gamma_{jm}\right)$$

$$\gamma_{jm} = \frac{\left|\mathbb{E}\left[\sqrt{\lambda_j}\mathbf{h}_{jjm}^{\mathsf{H}}\mathbf{w}_{jm}\right]\right|^2}{\frac{1}{\rho} + \mathrm{var}\left[\sqrt{\lambda_j}\mathbf{h}_{jjm}^{\mathsf{H}}\mathbf{w}_{jm}\right] + \sum_{(l,k)\neq(j,m)} \mathbb{E}\left[\left|\sqrt{\lambda_l}\mathbf{h}_{ljm}^{\mathsf{H}}\mathbf{w}_{lk}\right|^2\right]}.$$

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Two specific precoders $\mathbf{W}_j$:

$$\mathbf{W}_j^{\mathsf{BF}} \triangleq \hat{\mathbf{H}}_{jj}$$

$$\mathbf{W}_j^{\mathsf{RZF}} \triangleq \left(\hat{\mathbf{H}}_{jj}\hat{\mathbf{H}}_{jj}^{\mathsf{H}} + \mathbf{F}_j + N\alpha\mathbf{I}_N\right)^{-1}\hat{\mathbf{H}}_{jj}$$

where $\alpha > 0$ and $\mathbf{F}_j$ are design parameters.

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Large system analysis based on random matrix theory

Assume $N, K \to \infty$ at the same speed. Then,

$$\gamma_{jm} - \bar{\gamma}_{jm} \xrightarrow{\text{a.s.}} 0$$

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$$\bar{\gamma}_{jm}^{\text{RZF}} = \frac{\bar{\lambda}_j \delta_{jm}^2}{\frac{K}{N\rho}\left(1+\delta_{jm}\right)^2 + \frac{1}{N}\sum_{l,k}\bar{\lambda}_l \left(\frac{1+\delta_{jm}}{1+\delta_{lk}}\right)^2 \mu_{ljmk} + \sum_{l\neq j}\bar{\lambda}_l \left(\frac{1+\delta_{jm}}{1+\delta_{lm}}\right)^2 \left|\vartheta_{ljm}\right|^2}$$

and $\bar{\lambda}_j$, δ_{jm}, μ_{jlkm} and ϑ_{jlm} can be calculated numerically.

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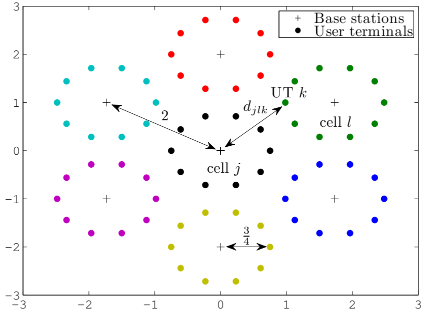
Downlink: Numerical results



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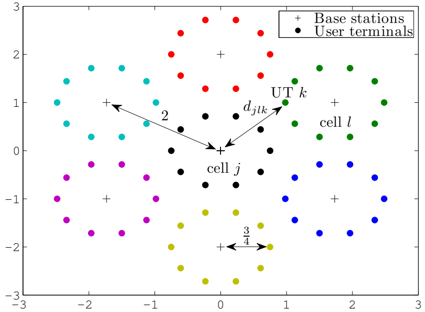

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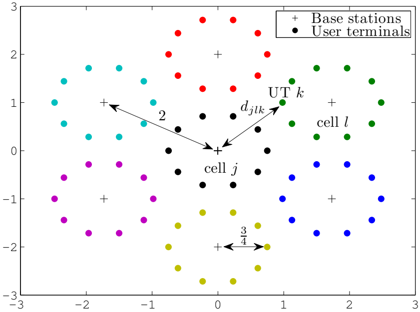

- 7 cells, $K = 10$ UTs distributed on a circle of radius $3/4$
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- Two channel models:
  - No correlation
  - $\tilde{\mathbf{R}}_{jlk} = d_{jlk}^{-\beta/2} [\mathbf{A}\ \mathbf{0}_{N \times N-P}]$, where $\mathbf{A} = [\mathbf{a}(\phi_1) \cdots \mathbf{a}(\phi_P)] \in \mathbb{C}^{N \times P}$ with

$$\mathbf{a}(\phi_p) = \frac{1}{\sqrt{P}} \left[1, e^{-\mathrm{i}2\pi c \sin(\phi)}, \ldots, e^{-\mathrm{i}2\pi c (N-1) \sin(\phi)}\right]^{\mathsf{T}}$$

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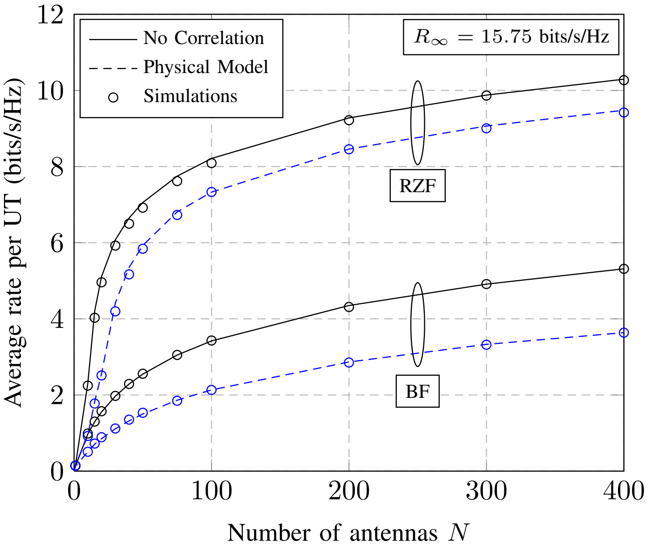

$$P = N/2, \alpha = 1/\rho, \mathbf{F}_j = \mathbf{0}$$

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- For finite N, RZF is largely superior to BF:
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Related work:

- Overview paper: Rusek, et al., "Scaling up MIMO: Opportunities and Challenges with Very Large Arrays", IEEE Signal Processing Magazine, to appear. http://liu.diva-portal.org/smash/record.jsf?pid=diva2:450781
- Constant-envelope precoding: S. Mohammed, E. Larsson, "Single-User Beamforming in Large-Scale MISO Systems with Per-Antenna Constant-Envelope Constraints: The Doughnut Channel", http://arxiv.org/abs/1111.3752v1
- Network MIMO TDD systems: Huh, Caire, et al., "Achieving "Massive MIMO" Spectral Efficiency with a Not-so-Large Number of Antennas", http://arxiv.org/abs/1107.3862

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- Full-duplex radios for cellular communications?

Related publications

- T. L. Marzetta
  Noncooperative cellular wireless with unlimited numbers of base station antennas
  IEEE Trans. Wireless Commun., vol. 9, no. 11, pp. 3590–3600, Nov. 2010.
- H. Q. Ngo, E. G. Larsson, T. L. Marzetta
  Analysis of the pilot contamination effect in very large multicell multiuser MIMO systems for physical channel models
  Proc. IEEE SPAWC'11, Prague, Czech Repulic, May 2011.
- H. Q. Ngo, E. G. Larsson, T. L. Marzetta
  Uplink power efficiency of multiuser MIMO with very large antenna arrays
  Allerton Conference, Urbana-Champaing, Illinois, US, Sep. 2011.
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Thank you!