Large System Analysis for Green Communications

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SUPELEC

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Outline

- SPACE: Random Matrix Theory and Stochastic Geometry
  - State-of-the-Art: RMT and SG for Multi-cell processing
  - Deterministic equivalents: From fixed to random user locations
  - A one-dimensional toy model
- TIME: Mean Field Games
  - Introduction to the stochastic time model
  - Toy model: optimal packet transmission time

SPACE: Random Matrix Theory and Stochastic Geometry

Motivation

- Cellular networks become increasingly dense: More antennas per m^2 (MIMO, small cells, femto cells, etc.)
- This implies a larger and larger energy footprint
- It is interesting to ask: How to densify?
  More antennas per base station (BS), more BSs?
- Can we compensate for densification by coordination?
  (multi-cell processing, interference coordination)

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- Can we compensate for densification by coordination? (multi-cell processing, interference coordination)

These are difficult questions, since one needs to account for:

- fading channels
- path loss
- random user and possibly BS locations
- cell/cluster association
- inter-/intra-cell interference
- imperfect channel state information and limited backhaul capacity
- different transmit/receive/cooperation strategies

State-of-the-art

Stochastic geometry for cellular (cooperative) systems:

- J. G. Andrews, F. Baccelli, and R. K. Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks", IEEE Trans. Comm., Nov. 2011.
- K. Huang and J. G. Andrews, "A Stochastic-Geometry Approach to Coverage in Cellular Networks With Multi-Cell Cooperation", IEEE Globecom, December 2011.
- K. Huang and J. G. Andrews, "Characterizing Multi-Cell Cooperation via the Outage-Probability Exponent:, submitted to IEEE ICC, Jun. 2012.

→ Main Focus: downlink, outage probability of a typical UT

→ Enables the study of heterogeneous, randomly deployed networks.

→ Only *interference coordination*, no joint transmissions, random BS-clustering

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Random matrix theory for multi-cell cooperative systems:

- D. Aktas, M. Bacha, J.Evans, S. Hanly, "Scaling Results on the Sum Capacity of Cellular Networks with MIMO Links, IEEE Trans. Inf. Theory, Jul. 2006
- H. Huh, A. Tulino, G. Caire, "Network MIMO with Linear Zero-Forcing Beamforming: Large System Analysis, Impact of Channel Estimation and Reduced-Complexity Scheduling", arxiv: http://arxiv.org/pdf/1012.3198
- J. Hoydis, M. Kobayashi, M. Debbah, "Optimal Channel Training in Uplink Network MIMO Systems:, IEEE. Trans. Sig. Proc., May 2011.

$\rightarrow$ Main focus: mutual information, achievable rates with linear receivers/decoders

$\rightarrow$ Account for realistic impairments, e.g. imperfect CSI, limited backhaul capacity

$\rightarrow$ Provides a "deterministic abstraction" of the physical layer.

$\rightarrow$ Only *fixed topology*

Objective

- Stochastic geometry is a powerful tool to study cellular networks with random user/access point distributions. But the application to multi-cell cooperative systems seems difficult.

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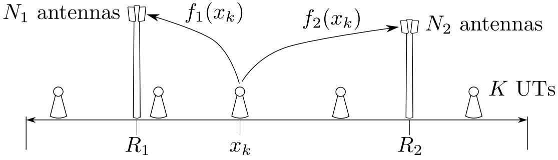
Can we combine both tools?

Important because we want to know, for a given probabilistic user distribution:

- Which BSs should cooperate?
- How much can we gain from cooperation?
- Where to place the BSs?

Deterministic equivalents: Cooperation with fixed user terminals



$$\mathbf{y} = \begin{pmatrix} \mathbf{y}_1 \\ \vdots \\ \mathbf{y}_B \end{pmatrix} = \mathbf{H}\mathbf{s} + \mathbf{n} = \begin{pmatrix} \mathbf{G}_1 \mathbf{T}_1^{\frac{1}{2}} \\ \vdots \\ \mathbf{G}_B \mathbf{T}_B^{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_K \end{pmatrix} + \begin{pmatrix} \mathbf{n}_1 \\ \vdots \\ \mathbf{n}_B \end{pmatrix}$$

- $s_k \sim \mathcal{CN}(0, \rho)$: transmit symbol of UT K
- $\mathbf{n}_b \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{N_b})$: noise at BS b
- $\mathbf{G}_b \in \mathbb{C}^{N_b \times K}$, $[\mathbf{G}_b]_{i,j} \sim \mathcal{CN}\left(0, \frac{1}{K}\right)$: fast fading
- $\mathbf{T}_b = \operatorname{diag}\left(f_b(x_k)\right)_{k=1}^{K}$ where $f_b(x)$ is a path loss function, e.g.

$$f_b(x) = \frac{1}{(1 + |R_b - x|)^{\beta}}$$

Deterministic equivalents: Mutual information and MMSE sum-rate

Theorem (Hachem, AAP'07)

Denote $c = \frac{N}{K}$, $c_i = \frac{N_i}{K}$ $\forall i$. *For* $N_i, K \to \infty$ *at the same speed,*

$$\frac{1}{N} \log det\left(\mathbf{I}_N + \rho \mathbf{H}\mathbf{H}^{\mathrm{H}}\right) - \bar{V}_N(\rho) \xrightarrow{a.s.} 0$$

where

$$\bar{V}_N(\rho) = \sum_{i=1}^{B} c_i \log\left(\frac{\rho}{\Psi_i}\right) + \frac{1}{N}\sum_{k=1}^{K} \log\left(1 + \sum_{i=1}^{B} c_i f_i(x_k)\Psi_i\right) - \frac{1}{N}\sum_{k=1}^{K} \frac{\sum_{i=1}^{B} c_i f_i(x_k)\Psi_i}{1 + \sum_{i=1}^{B} c_i f_i(x_k)\Psi_i}$$

and $\Psi_1, \ldots, \Psi_B$ *are given as the unique positive solution to*

$$\Psi_i = \left(\frac{1}{\rho} + \frac{1}{K}\sum_{k=1}^{K} \frac{f_i(x_k)}{1 + \sum_{i=1}^{B} c_i f_i(x_k)\Psi_i}\right)^{-1}, \qquad i = 1, \ldots, B.$$

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Remark

SINR with MMSE detection: $\gamma_k = \mathbf{h}_k^{\mathsf{H}} \left(\mathbf{H}\mathbf{H}^{\mathsf{H}} - \mathbf{h}_k \mathbf{h}_k^{\mathsf{H}} + \frac{1}{\rho} \mathbf{I}_N \right)^{-1} \mathbf{h}_k \asymp \sum_{i=1}^{B} c_i f_i(x_k) \Psi_i$.

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Thus: $R_{\text{sum}} = \frac{1}{N} \sum_{k=1}^{K} \log(1 + \gamma_k) \asymp \frac{1}{N} \sum_{k=1}^{K} \log \left(1 + \sum_{i=1}^{B} c_i f_i(x_k) \Psi_i \right)$.

Deterministic equivalents: Random user locations

Assume that the positions x_k of the UTs are i.i.d. with distribution F. Then,

$$\frac{1}{N}\sum_{k=1}^{K}\log\left(1+\sum_{i=1}^{B}c_i f_i(x_k)\Psi_i\right)\approx\frac{1}{c}\int\log\left(1+\sum_{i=1}^{B}c_i f_i(x)\Psi_i\right)dF(x).$$

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Similarly,

$$\Psi_i=\left(\frac{1}{\rho}+\frac{1}{K}\sum_{k=1}^{K}\frac{f_i(x_k)}{1+\sum_{i=1}^{B}c_i f_i(x_k)\Psi_i}\right)^{-1}\approx\left(\frac{1}{\rho}+\int\frac{f_i(x)}{1+\sum_{i=1}^{B}c_i f_i(x)\Psi_i}dF(x)\right)^{-1}.$$

Deterministic equivalents: Random user locations

Corollary

Let x_k, $k = 1, \ldots, K$, be i.i.d. with distribution F. Then,

$$\frac{1}{N} \log\det\left(\mathbf{I}_N + \rho \mathbf{H}\mathbf{H}^{\mathrm{H}}\right) - \bar{I}_N(\rho) \xrightarrow{\text{a.s.}} 0$$

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where $\psi_1, \ldots, \psi_B$ are given as the unique positive solution to

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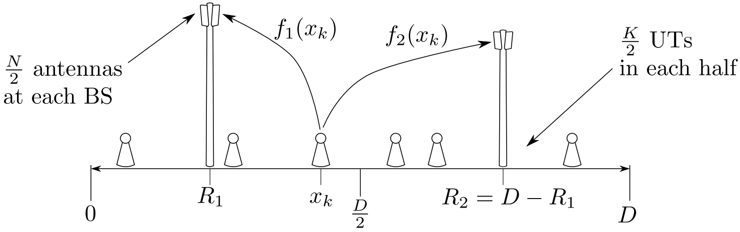
Application: Optimal BS-placement

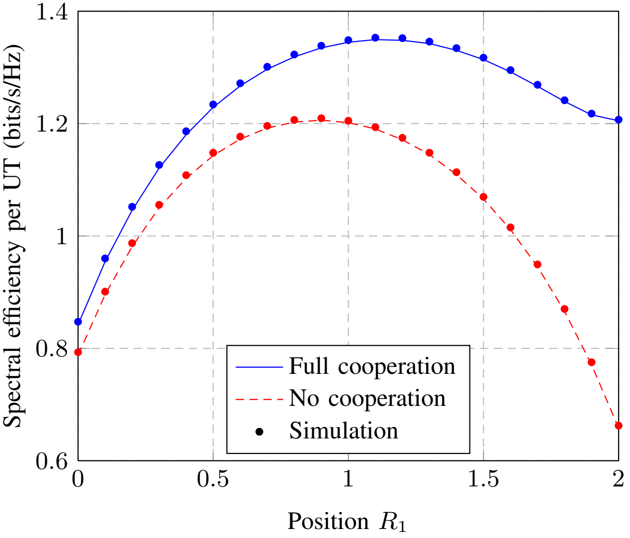


- $\frac{K}{2}$ UTs uniformly distributed on the intervals $[0, \frac{D}{2}]$ and $[\frac{D}{2}, D]$, respectively.
- Path loss functions: $f_i(x) = (1 + |R_i - x||)^{-\beta}, \qquad i = 1, 2.$
- Decompose the channel matrix as $\mathbf{H} = \begin{pmatrix} \mathbf{H}_{1,1} & \mathbf{H}_{1,2} \\ \mathbf{H}_{2,1} & \mathbf{H}_{2,2} \end{pmatrix}$, where $\mathbf{H}_{i,j} \in \mathbb{C}^{N/2 \times K/2}$.
- Mutual information without cooperation:

$$I_N^{\mathrm{nc}}(\rho) = \frac{1}{N} \sum_{i=1}^{2} \log\det\left(\mathbf{I}_{N/2} + \rho \mathbf{H}_{i,i}\mathbf{H}_{i,i}^{\mathrm{H}} + \rho \mathbf{H}_{i,\bar{i}}\mathbf{H}_{i,\bar{i}}^{\mathrm{H}}\right) - \log\det\left(\mathbf{I}_{N/2} + \rho \mathbf{H}_{i,\bar{i}}\mathbf{H}_{i,\bar{i}}^{\mathrm{H}}\right)$$

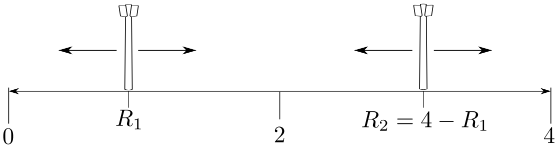
where $\bar{i} = 1 + i \bmod 2$.

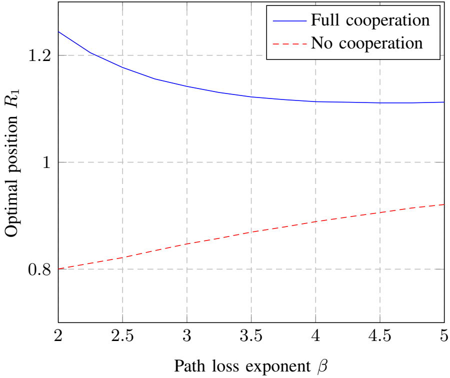
Optimal BS-placement: Numerical results (I)

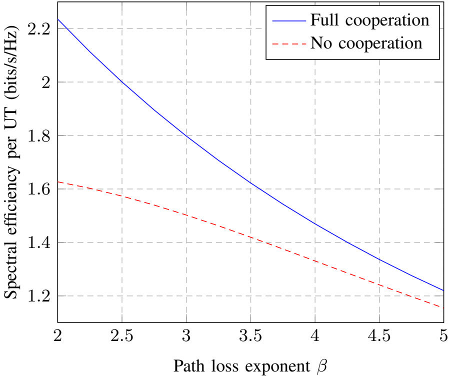


$N = 16,\ K = 12,\ \rho = 10\,\text{dB},\ \beta = 3.7,\ D = 4$

Optimal BS-placement: Numerical results (II)







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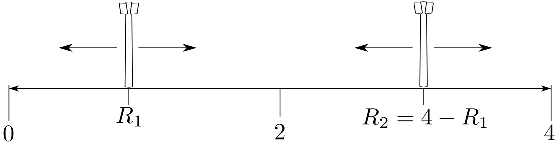

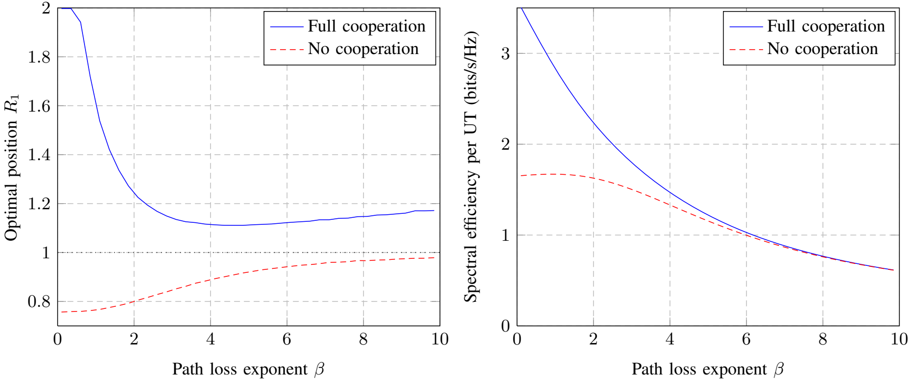

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Some remarks

- A combination of RMT and stochastic geometry is possible but so far on simplistic models!
- Asymptotic results are accurate for realistic (large) system dimensions.
- We can optimize system parameters with respect to random channel realizations and user distributions, without simulations.
- The same results could be also applied for MMSE/MRC detectors.
- We can also account for imperfect CSI, limited backhaul capacity.
- Extensions to two-or three-dimensional models are possible.
- *Keep in mind that the BS-positions are deterministic!*

TIME: Mean Field Games

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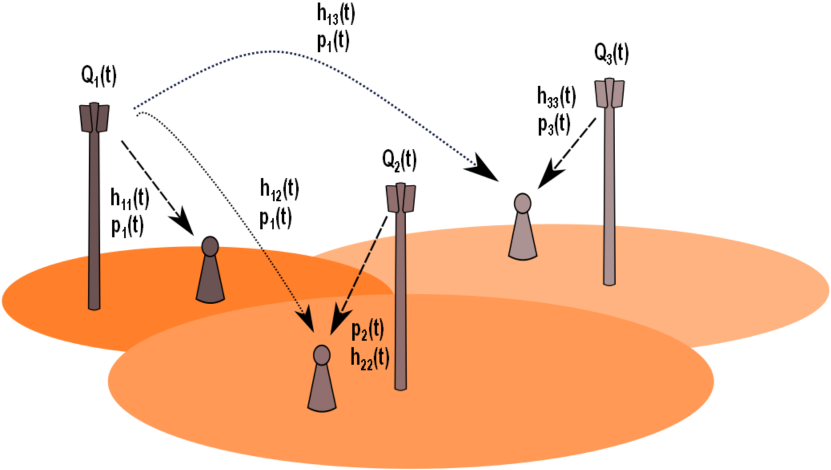
- Assuming latency is allowed, time multiplexing can be used as well
- We explore here the behaviour of a decentralized system minimizing energy over time while ensuring data arrival at deadline

Decentrliazed time scheduling

We wish to determine a downlink strategic distribution of consumed power over time

- that minimizes the individual BS power consumption
- that ensures final arrival of the expected data
- under light assumptions on the knowledge about adjacent cells

System description



Scenario and objectives (1/2)

We consider:

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- A network of N BSs and N UTs
- UT l receives from BS l a packet within a time window T, with initial size $Q_l(0)$
- BS l chooses its power policy $\{p_l(t)\}_{t\in[0,T]}$
- BSs interfere adjacent cell users
- The channel gains $h_{nl}(t) > 0$ from BS n to UT l are time-varying
- The data size $Q_l(t)$ is evolves with SINR at the receiver

$$dQ_l(t) = -B\log(1+\mathrm{SINR}_l(t)), \quad \mathrm{SINR}_l(t) = \frac{p_l(t)h_{ll}(t)}{\sigma_l^2 + \sum_{j\neq l} h_{jl}(t)p_j(t)}$$

The mean field game framework (1/2)

- We assume a decentralized selfish optimization based on successively sensed data and initial prior knowledge
- Hence, we consider a game theoretical approach, with N players, the BS/UT couples.
  - each player establishes a power control strategy so to minimize its cost under termination constraint
  - each player reacts to changes in other player's actions

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- Hence, we consider a game theoretical approach, with N players, the BS/UT couples.
  - each player establishes a power control strategy so to minimize its cost under termination constraint
  - each player reacts to changes in other player's actions
- However, N-body differential games are difficult to solve, as soon as $N > 1$.

The mean field game framework (2/2)

- Mean Field Games (MFG) simplify these games by assuming $N \to \infty$ and a lot of symmetry in the system
  - players individual actions do *not* impact overall behavior
  - overall behavior led by the mass of all players.

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- Mean Field Games (MFG) simplify these games by assuming $N \to \infty$ and a lot of symmetry in the system
  - players individual actions do *not* impact overall behavior
  - overall behavior led by the mass of all players.
- BS's state variables $Q_1(t), \ldots, Q_N(t)$ are turned into a density $m(t, Q)$

$$m(t, Q)dQ \simeq \frac{1}{N} \sum_{n=1}^{N} \delta_{Q \leq Q_n(t) \leq Q+dQ} dQ.$$

- BS's power strategy $p(t, Q)$ is a reaction to $m(t, Q)$.

MFG definition

In order to define an MFG, some simplifying assumptions will be used:

- The interference term

$$I(t) = \sum_{k \neq l} h_{kl}(t) p_k(t)$$

needs a mean-field limit. We choose an appropriate scaling by γ/N for some $\gamma > 0$ constant

$$I^{\infty}(t) = \gamma \int_Q \int_h m(t,Q,h) h(t) p(t,Q,h) dQdh$$

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- For readability (although not necessary), we take $h(t) = 1$ constant. (otherwise, we would consider the dual-state variable (Q, h))

MFG equations

The optimization problem we consider is the following:

$$\min_{p(t)} \int_0^T p(t)dt + K(Q(T))$$

with $K(Q)$ a terminal cost function, such that

$$dQ(t) = -B\log(1 + \mathrm{SINR}(t, m_t, p(t)))$$

where,

$$\mathrm{SINR}(t, m_t, p(t)) = \frac{p(t)}{\sigma^2 + I^\infty(t, m_t)}$$

for given $Q(0)$ and m_0.

MFG equations

We consider the running cost function $v(t, Q)$

$$v(t,Q) = \int_t^T p(u)du + K(Q(T))$$

An optimal power control $p^*(t, Q)$ exists if there exists a function $v^*(t, Q)$ solution to the Hamilton-Jacobi-Bellman equation:

$$\partial_t v(t,Q) + \inf_{\tilde{p}(t)} [\tilde{p}(t) - B\log(1 + \mathrm{SINR}(t, m_t^*, \tilde{p}(t)))\partial_Q v(t,Q)] = 0$$

where m_t^* is solution to the Fokker-Planck equation

$$\partial_t m_t - B\partial_Q \log(1 + \mathrm{SINR}(t, m_t, p^{(v^*)}(t))m_t] = 0$$

with $p^{(v^*)}(t) = p(t)$ when $v = v^*$. This is precisely

$$\partial_t m_t = B\log\left(\frac{B\partial_Q v^*(t,Q)}{\sigma^2 + I^\infty(t, m_t)}\right)\partial_Q m_t + m_t \frac{\partial^2_{QQ} v^*(t,Q)}{\partial_Q v^*(t,Q)}.$$

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This defines a system of coupled partial differential equation.

From the solutions above, we finally obtain

$$p^*(t,Q) = -(\sigma^2 + I^\infty(m_t^*)) + B\partial_Q v^*(t,Q)$$

Discussion on the framework

- The MFG assumption result has important consequences on the results:
  - solution trajectory defined entirely from initial state $m(0, \cdot)$
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  - $\Rightarrow$ BSs can run on the mean field equilibrium from $t = 0$
  - averaged interference (not a single interference but a diffuse set) is a strong assumption
  - $\Rightarrow$ MFG only capture a high level vision

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  - solution trajectory defined entirely from initial state $m(0, \cdot)$
  - $\Rightarrow$ BSs can run on the mean field equilibrium from $t = 0$
  - averaged interference (not a single interference but a diffuse set) is a strong assumption
  - $\Rightarrow$ MFG only capture a high level vision
- Extensions to more realistic scenarios are possible (e.g. annuli of interference) but differential equations will no longer be solvable.

Simulation results (1/3)

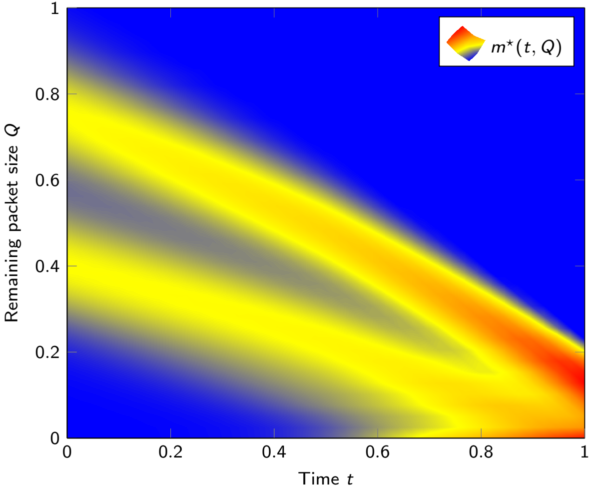

Figure: Optimal distribution of users $m^{*}(t, Q)$

Simulation results (2/3)

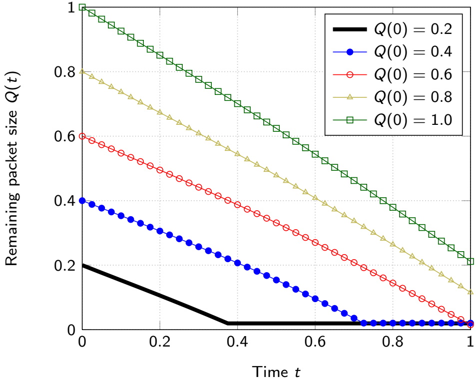

Figure: Remaining packet size $Q(t)$ under optimal power $p^*(t, Q_t)$ for users with initial packet size $Q(0) \in \{0.2, 0.4, 0.6, 0.8, 1\}$.

Simulation results (3/3)

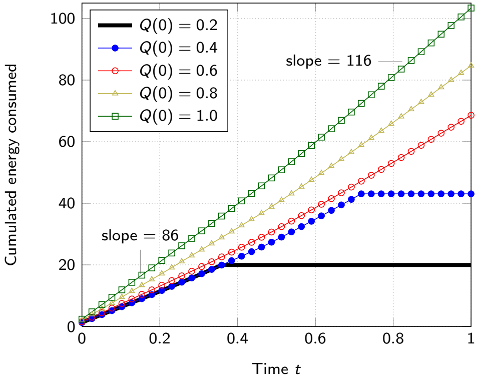

Figure: Cumulated energy consumption $\int_0^t p^*(u, Q(u))du$ under optimal power policy $p^*(t, Q)$ for users with initial packet sizes $Q(0) \in \{0.2, 0.4, 0.6, 0.8, 1\}$.

General Conclusions

- Large dimensional analysis tools allow multiple system abstractions:
  - abstraction of fast fading matrices (PHY) with RMT
  - abstraction of user's locations (MAC) with MFG

$\Rightarrow$ These lead to model simplifications to tackle hard problems involving
  - multiple cells and users
  - cooperation, backhaul link limitations
  - time/delay constraints,...

General Conclusions

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  $\Rightarrow$ These lead to model simplifications to tackle hard problems involving
  - multiple cells and users
  - cooperation, backhaul link limitations
  - time/delay constraints,...
- Green communications challenges and open questions can be explored within these frameworks

  Here we considered space and time diversity optimization

Thank you!