Outage Performance of Cooperative Small Cell Systems Under Rician Fading Channels

Supélec

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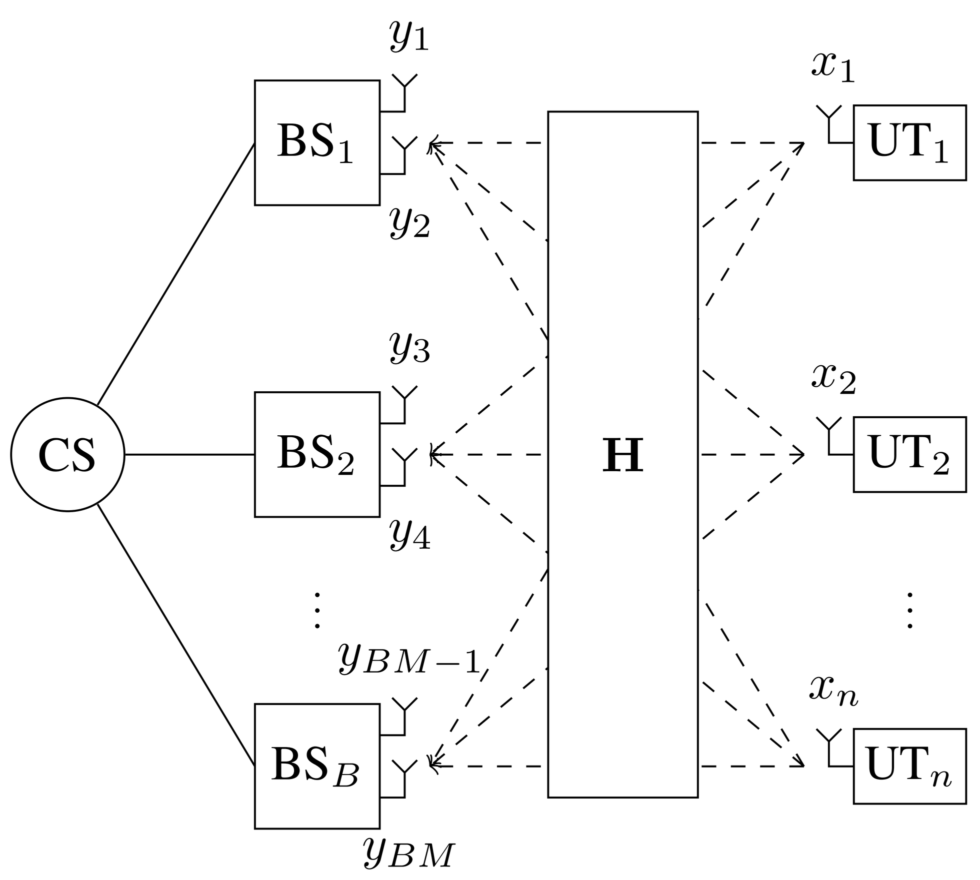
1 Abstract

- Rician fading multiple-input multiple-output (MIMO) channel with a variance profile
- Relevant to cooperative small-cell systems where several densely deployed base stations (BSs) cooperatively serve multiple user terminals (UTs) [1]
- Large system analysis assuming many BSs or BS-antennas and UTs
- Central limit theorem (CLT) of the mutual information and explicit expression of the asymptotic variance
- Application: Approximation of the outage probability
- Asymptotic performance predictions are accurate for small channel dimensions

2 System model



- Uplink channel from n single-antenna UTs to B BSs with M antennas each
- BSs connected to a central station (CS) via infinite-capacity backhaul links
- CS jointly processes the signals from all BSs
- Full CSI at the CS

2.1 Uplink channel model

$N \times n$ MIMO channel from n user terminals (UTs) to a receiver with $N = BM$ distributed antennas:

$$\mathbf{y} = \sqrt{\rho}\mathbf{H}\mathbf{x} + \mathbf{n}$$

where

- $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_n)$: transmit vector
- $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$: noise
- $\mathbf{H} = [\mathbf{H}_1^\mathsf{T} \cdots \mathbf{H}_B^\mathsf{T}]^\mathsf{T} \in \mathbb{C}^{BM\times n}$: aggregate channel

Rician fading channel:

$$[\mathbf{H}_b]_{m,k} = \underbrace{\sqrt{\frac{(1-\kappa_{bk})\,\ell_{bk}}{n}}\,w_{mk}^b}_{\text{Rayleigh component}} + \underbrace{\sqrt{\frac{\kappa_{bk}\,\ell_{bk}}{n}}\,e^{j\phi_{mk}^b}}_{\text{LOS component}}$$

where

- $w_{mk}^b \sim \mathcal{CN}(0,1)$: fast fading
- ℓ_{bk}: inverse path loss
- $\phi_{mk}^b \in [0, 2\pi)$: phase of specular component
- $\kappa_{bk} \in [0,1]$: Rician parameter

Normalized mutual information:

$$\mathcal{I}(\rho) = \frac{1}{N}\log\det\left(\mathbf{I}_N + \frac{1}{\rho}\mathbf{H}\mathbf{H}^\mathsf{H}\right)$$

$$I(\rho) = \mathbb{E}\left(\mathcal{I}(\rho)\right)$$

3 Related results

Theorem 1 (Deterministic Equivalent) [2] *Under some mild technical assumptions:*

(i) The following set of $N+n$ deterministic equations,

$$\psi_i(z) = \frac{1}{\rho\left(1+\frac{1}{n}\mathrm{tr}\,\widetilde{\mathbf{D}}_i\widetilde{\mathbf{T}}(z)\right)}, \quad 1 \le i \le N$$

$$\widetilde{\psi}_j(z) = \frac{1}{\rho\left(1+\frac{1}{n}\mathrm{tr}\,\mathbf{D}_j\mathbf{T}(z)\right)}, \quad 1 \le j \le n$$

where

$$\mathbf{\Psi}(z) = \mathrm{diag}\left(\psi_i(z),\, 1 \le i \le N\right)$$
$$\widetilde{\mathbf{\Psi}}(z) = \mathrm{diag}\left(\widetilde{\psi}_j(z),\, 1 \le j \le n\right)$$
$$\mathbf{T}(z) = \left(\mathbf{\Psi}(z)^{-1} + \rho\mathbf{A}\widetilde{\mathbf{\Psi}}(z)\mathbf{A}^\mathsf{H}\right)^{-1}$$
$$\widetilde{\mathbf{T}}(z) = \left(\widetilde{\mathbf{\Psi}}(z)^{-1} + \rho\mathbf{A}^\mathsf{H}\mathbf{\Psi}(z)\mathbf{A}\right)^{-1}$$

admits a unique solution $(\psi_1(z), \dots, \psi_N(z), \widetilde{\psi}_1(z), \dots, \widetilde{\psi}_n(z)) \in \mathcal{S}^{N+n}$ for $z \in \mathbb{C}\setminus\mathbb{R}^+$.

(ii) Let $\rho > 0$ and consider the quantity:

$$V(\rho) = \frac{1}{N}\log\det\left(\frac{\mathbf{\Psi}(-\rho)^{-1}}{\rho} + \mathbf{A}\widetilde{\mathbf{\Psi}}(-\rho)\mathbf{A}^\mathsf{H}\right) + \frac{1}{N}\log\det\left(\frac{\widetilde{\mathbf{\Psi}}(-\rho)^{-1}}{\rho}\right) - \frac{\rho}{Nn}\sum_{\substack{i=1,\dots,N\\ j=1,\dots,n}}\sigma_{ij}^2\mathbf{T}_{ii}(-\rho)\widetilde{\mathbf{T}}_{jj}(-\rho)\,.$$

Then, the following holds true:

$$I(\rho) - V(\rho) \xrightarrow[N,n\to\infty]{} 0\,.$$

4 The central limit theorem

Theorem 2 (The CLT) Under some mild technical assumptions, the mutual information $\mathcal{I}(\rho)$ satisfies

$$\frac{N}{\Theta_{N,n}}\left(\mathcal{I}(\rho) - V(\rho)\right) \xrightarrow[N,n\to\infty]{\mathcal{D}} \mathcal{N}(0,1)$$

where $\mathcal{D}$ denotes convergence in distribution and the asymptotic variance $\Theta_{N,n}$ is given as

$$\Theta_{N,n}^2 = -\log\det(\mathbf{J})\,.$$

Letting $\mathbf{a}_i$, $\mathbf{b}_i$ $\mathbf{t}_i$ and $\widetilde{\mathbf{t}}_i$ denote respectively the columns of $\mathbf{A}$, $\mathbf{A}^\mathsf{H}$, $\mathbf{T}$ and $\widetilde{\mathbf{T}}$, the matrix $\mathbf{J}$ takes the following form:

$$\mathbf{J} = \begin{pmatrix}\mathbf{J}_1 & \mathbf{J}_2\\ \mathbf{J}_3 & \mathbf{J}_4\end{pmatrix}$$

where

$$[\mathbf{J}_1]_{\substack{k=1,\dots,n\\ m=1,\dots,n}} = 1_{\{k,m\}} - \frac{1}{n(1+\delta_m)^2}\mathbf{a}_m^\mathsf{H}\mathbf{T}\mathbf{D}_k\mathbf{T}\mathbf{a}_m$$
$$[\mathbf{J}_2]_{\substack{k=1,\dots,n\\ m=1,\dots,N}} = \frac{\rho}{n}\mathbf{t}_m^\mathsf{H}\mathbf{D}_k\mathbf{t}_m$$
$$[\mathbf{J}_3]_{\substack{k=1,\dots,N\\ m=1,\dots,n}} = \frac{\rho}{n}\widetilde{\mathbf{t}}_m^\mathsf{H}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{t}}_m$$
$$[\mathbf{J}_4]_{\substack{k=1,\dots,N\\ m=1,\dots,N}} = 1_{\{k,m\}} - \frac{1}{n(1+\widetilde{\delta}_m)^2}\mathbf{b}_m^\mathsf{H}\widetilde{\mathbf{T}}\widetilde{\mathbf{D}}_k\widetilde{\mathbf{T}}\mathbf{b}_m$$

where $1_{\{k,m\}} = 1$ for $k = m$ and zero otherwise.

Remark 4.1 *The matrix $\mathbf{J}$ can be seen as the Jacobian of the fundamental equations in Theorem 1 (i). This simple expression of the asymptotic variance is expected to hold for even more involved random matrix models.*

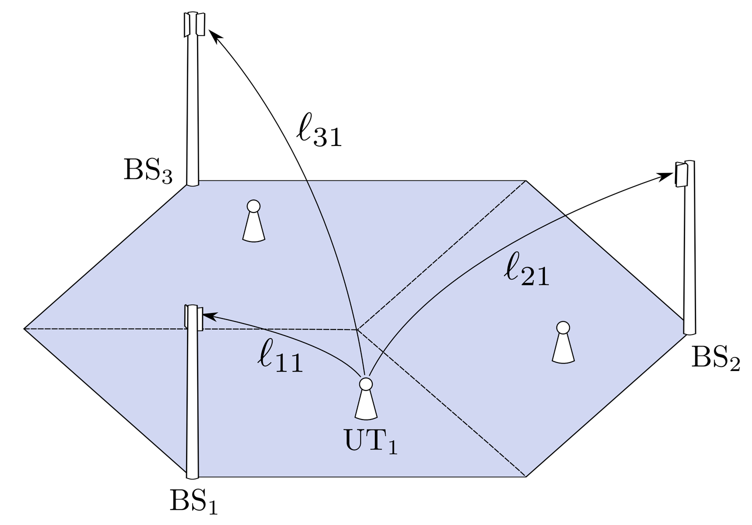
5 Outage probability

The CLT can be used to calculate an approximation of the outage probability:

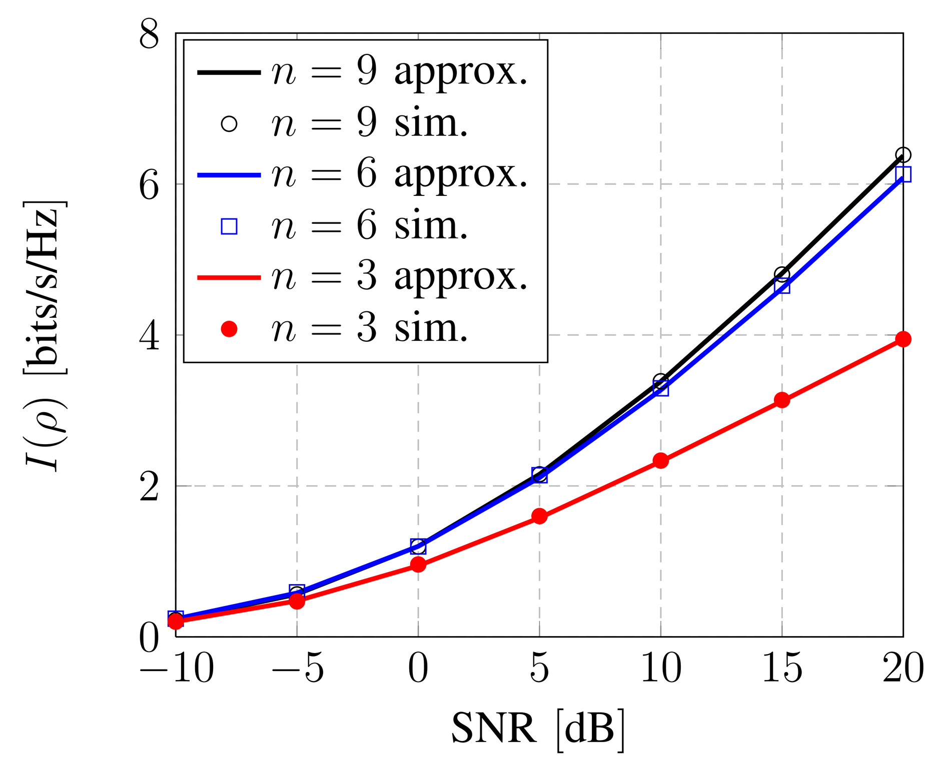
$$P_{\text{out}}(R) \triangleq \Pr(N\mathcal{I}(\rho) < R) \approx 1 - Q\left(\frac{R - NV(\rho)}{\Theta_{N,n}}\right)$$

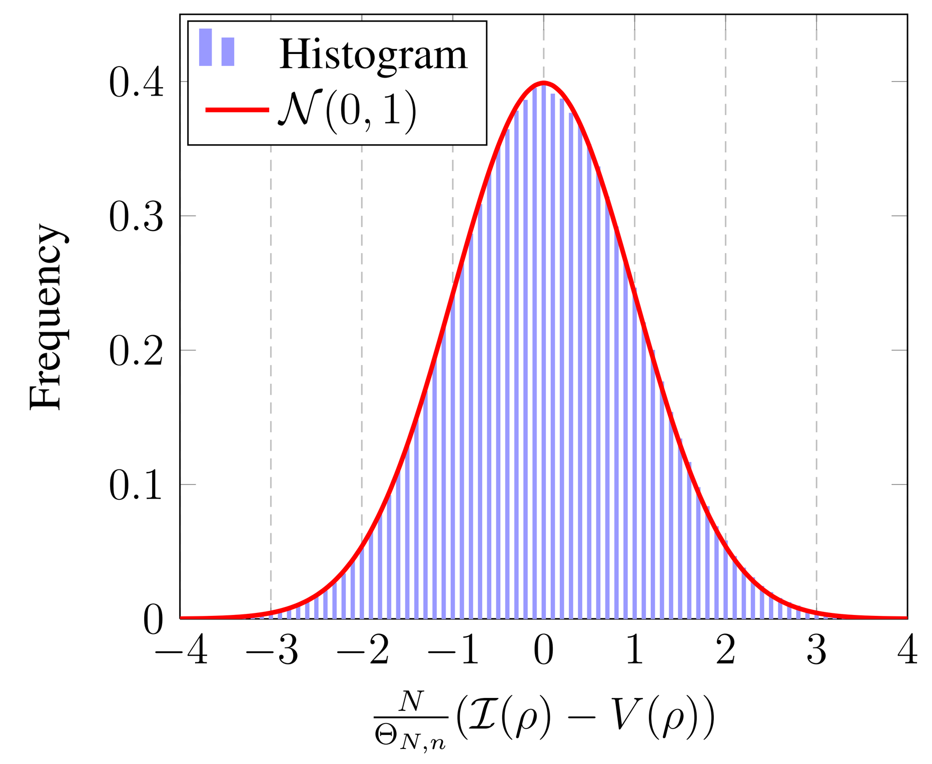
where $Q(x)$ is the Gaussian tail function.

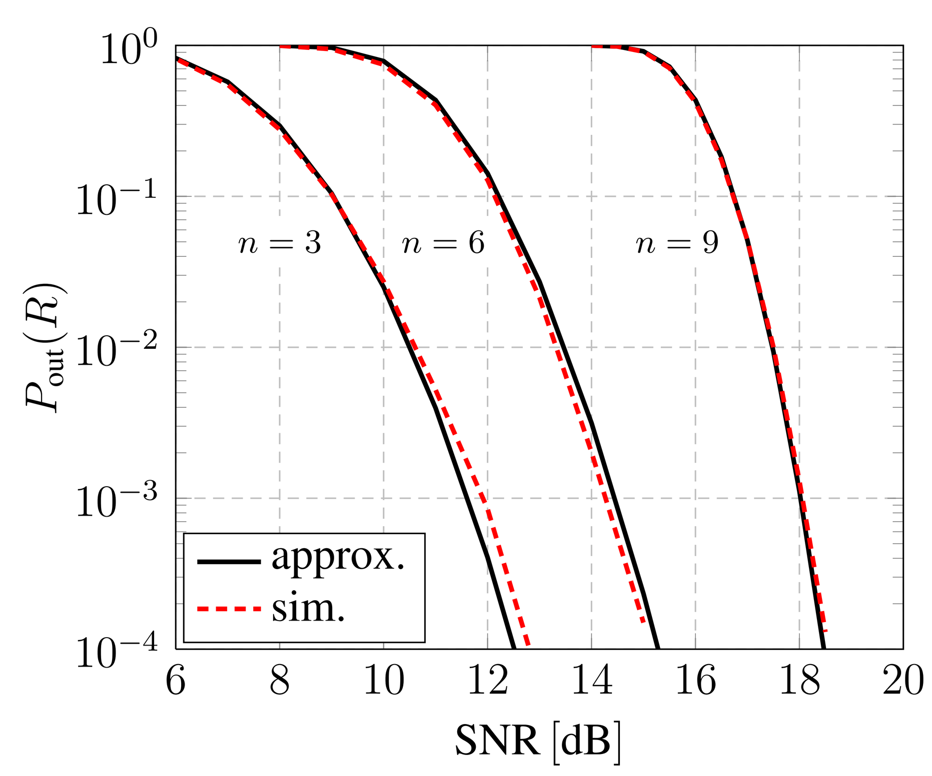
6 A cellular example



- $B = 3$ BSs with $M = 2$ antennas, i.e. $N = 6$
- $n = \{3, 6, 9\}$ UTs uniformly distributed over three cells
- Path loss exponent $\beta = 3.6$
- target rate $R = n \times 3$[nats/s/Hz]







7 References

1. J. Hoydis, M. Kobayashi, M. Debbah, "Green Small-Cell Networks," IEEE Veh. Technol. Mag., vol. 6, no. 1, Mar. 2011.
2. W. Hachem, P. Loubaton, J. Najim, "Deterministic Equivalents for Certain Functionals of Large Random Matrices," Annals of Applied Probability, vol. 17, no. 3, pp. 875930, 2007.
3. W. Hachem, P. Loubaton, J. Najim, "A CLT for Information-Theoretic Statistics of Gram Random Matrices with a Given Variance Profile," Annals of Applied Probability, vol. 18, no. 6, pp. 2071–2130, 2008.

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